Il codice usato per la presentazione è disponibile sulla mia pagina GitHub (https://github.com/OttaviaE/irt).

Dalla parte degli item

Georg Rasch e i modelli dell'Item Response Theory

Ottavia M. Epifania

Georg Rasch

Dalla base

1. $1 + 2$
2. $\frac{2}{3} + \frac{3}{5}$
3. $2x^2 + 3x + 4 = 0$

$\frac{A_{soggetto}}{D_{item}}$:

- $A_{soggetto} > D_{item} \rightarrow > 1$
- $A_{soggetto} < D_{item} \rightarrow < 1$

Qualche trasformazione dopo…

$$P(x_{si} = 1|\theta_s, b_i) = \frac{exp(\theta_s - b_i)}{1 + exp(\theta_s - b_i)}$$

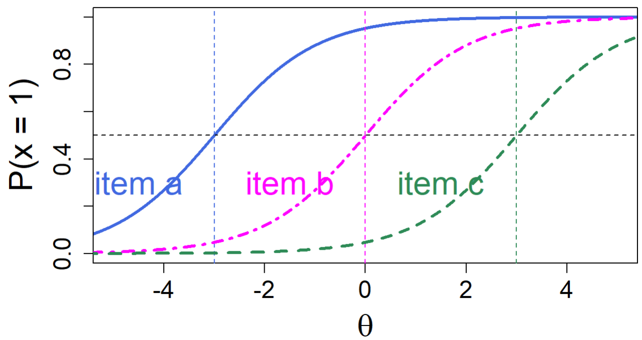
Generalized Linear Model

$logit = \ln\left(\frac{P(x=1)}{1-P(x=1)}\right)$

$P(x = 1) = \frac{\exp(\eta)}{1+exp(\eta)}$

where
$\eta = \alpha + \beta X$

θ_s, b_i e il tratto latente



Rasch in pillole

- I marginali di riga (colonna) sono statistiche sufficienti
- Sono i dati che si adattano al modello
- La triade:

1. Indipendenze locale
2. Oggettività specifica
3. Invarianza di misurazione

Item Response Theory

Così simili e così diversi

- I parametri degli item
- Il modello si adatta ai dati
- I marginali di riga (colonna) non sono più statistiche sufficienti

1Pl

- Confuso con il modello di Rasch
- Il parametro a che c'è e non c'è (dipende a chi chiedi)
- 1 parametro e mezzo

e.g. De Mars (2010)

$$P(x=1|\theta,b,a) = \frac{exp[a(\theta_s - b_i)]}{1+exp[a(\theta_s - b_i)]}$$

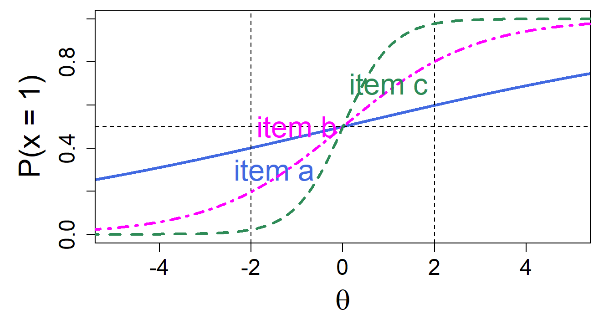
Da altre parti

$$P(x=1|\theta,b) = \frac{exp(\theta_s - b_i)}{1+exp(\theta_s - b_i)}$$

2PL (cambia la pendenza)

a (*discrimination*):

$$P(x=1|\theta,b,a) = \frac{exp[a_i(\theta_p - b_i)]}{1+exp[a_i(\theta_p - b_i)]}$$

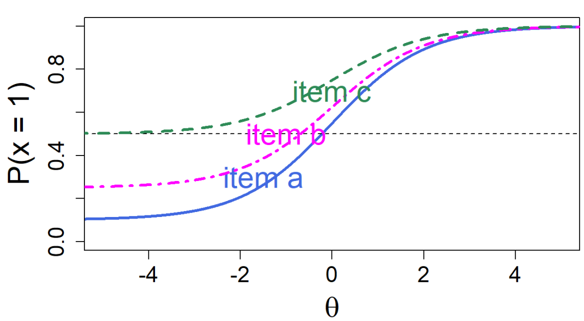


Spostando gli asintoti

3PL (alza in basso)

c (*lucky guess*):

$$P(x=1|\theta,b,a,c) = c_i + (1-c_i) + \frac{exp[a(\theta_s - b_i)]}{1+exp[a(\theta_s - b_i)]}$$

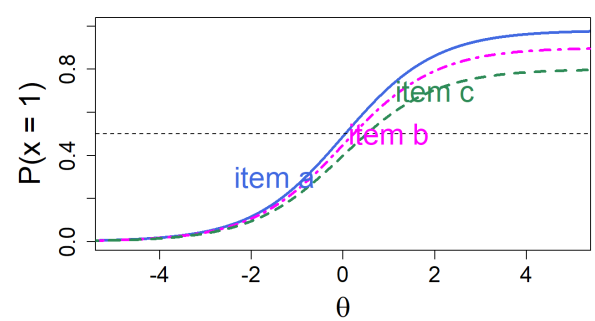


Spostando gli asintoti

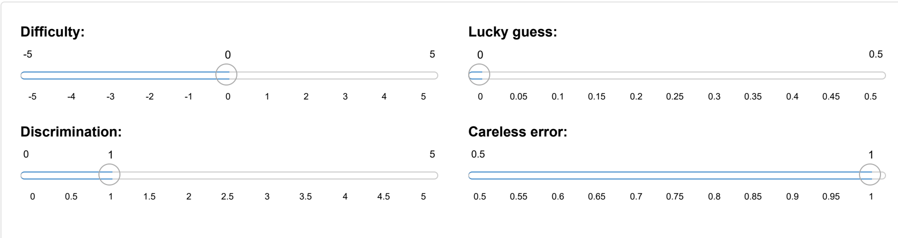
4PL (abbassa in alto)

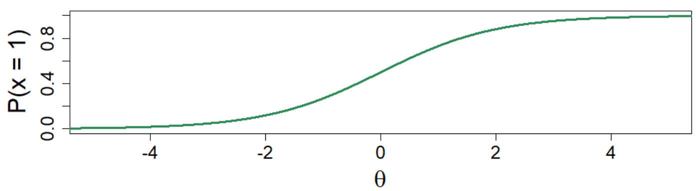
e (*careless error*):

$$P(x=1|\theta,b,a,c,e) = c_i + (e_i - c_i) + \frac{exp[a(\theta_s - b_i)]}{1+exp[a(\theta_s - b_i)]}$$



Tutti quanti



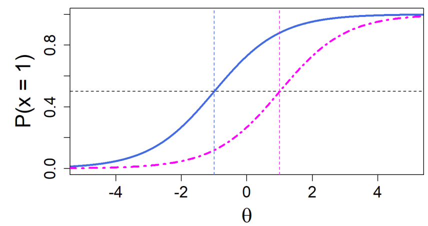


Differential Item Functioning

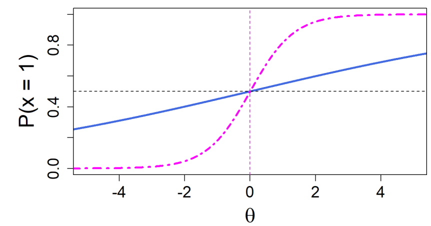
“Il cat calling è una molestia”:

- Si
- No

Uniforme



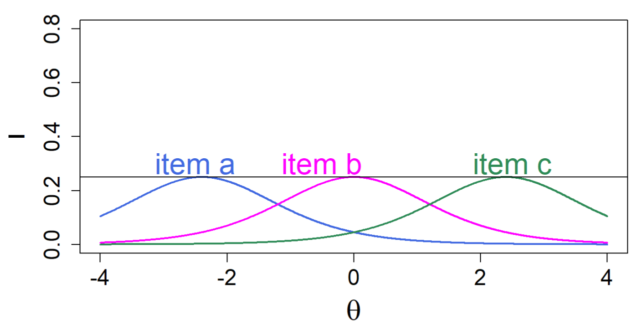
Non Uniforme



Item Information Function - Rasch

$$I_i(\theta) = P_i(\theta)(1 - P_i(\theta))$$

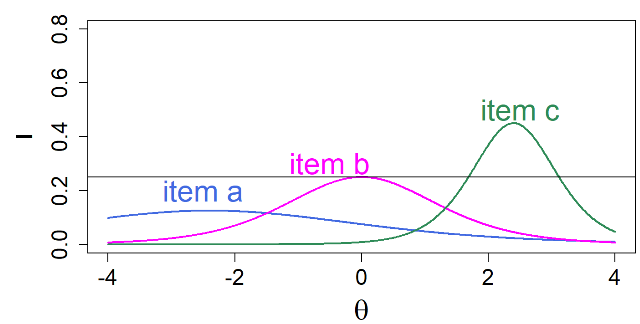
	item_a	item_b	item_c
difficulty	-3	0	3
discrimination	1	1	1



Item Information Function - 2PL

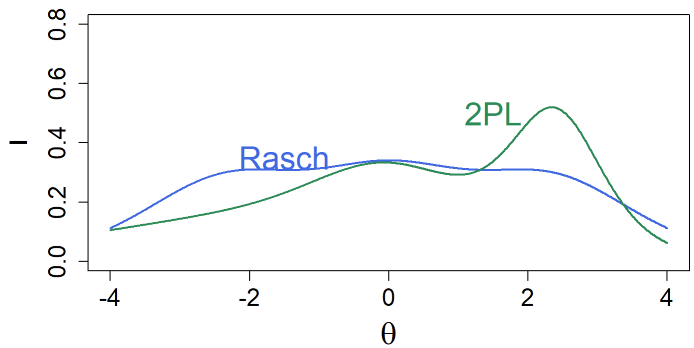
$$I_i(\theta) = a^2 P_i(\theta)(1 - P_i(\theta))$$

	item_a	item_b	item_c
difficulty	-3.0	0	3.0
discrimination	0.5	1	1.8



Test Information Function (TIF)

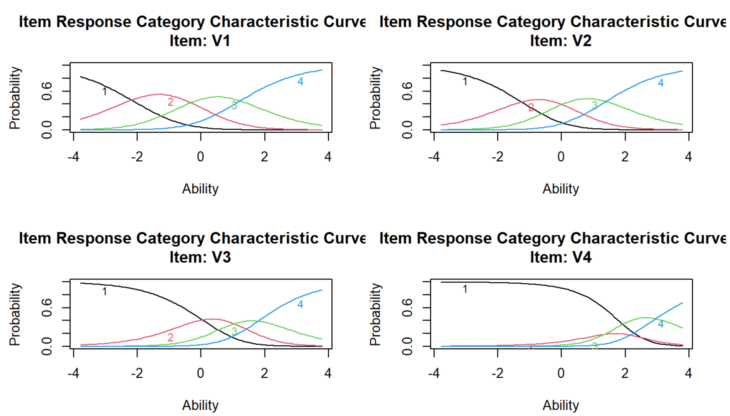
$$I(\theta) = \sum_{i=1}^{n} I_i(\theta)$$



Oltre il dicotomico

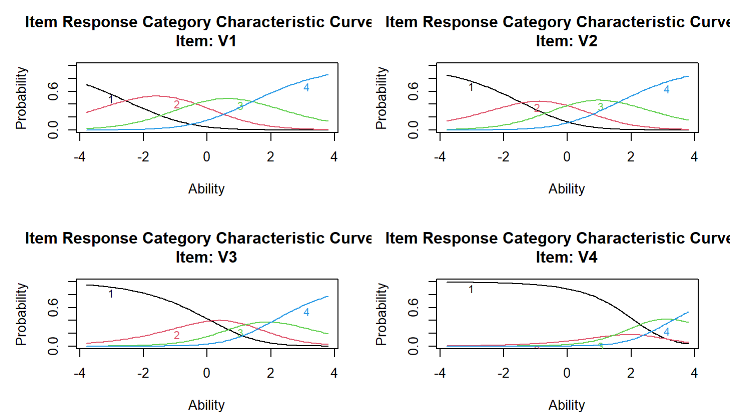
Polytomous Rasch Model

$$P(X = x, x > 0|\theta_s, \tau_{ic}) = \frac{exp(\sum_{c=1}^{m} \theta_s - \tau_{ic})}{1+exp(\sum_{c=1}^{m} \theta_s - \tau_{ic})}$$



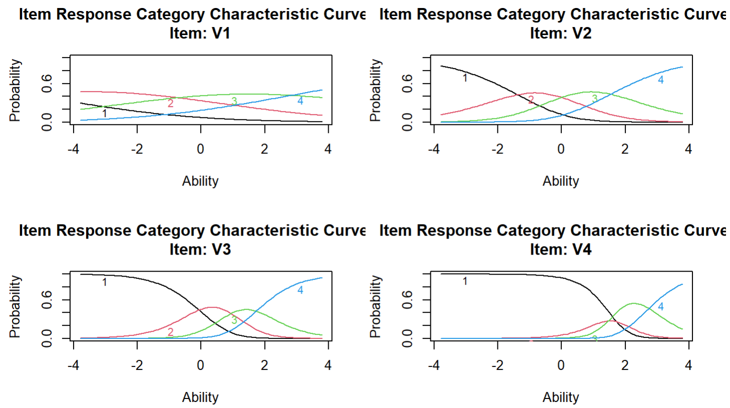
Partial credit model

$$P(X = x, x > 0|\theta_s, \tau_{ic}, a) = \frac{exp[\sum_{c=1}^{m} a(\theta_s - \tau_{ic})]}{1+exp[\sum_{c=1}^{m} a(\theta_s - \tau_{ic})]}$$

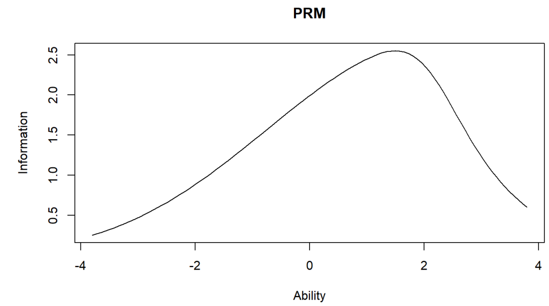


Graded Response Model

$$P(X = x, x > 0|\theta_s, \tau_{ic}, a_i) = \frac{exp[\sum_{c=1}^{m} a_i(\theta_s - \tau_{ic})]}{1+exp[\sum_{c=1}^{m} a_i(\theta_s - \tau_{ic})]}$$



PRM, PCM, GRM: TIF



Ackoejkl nkjgnkg

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