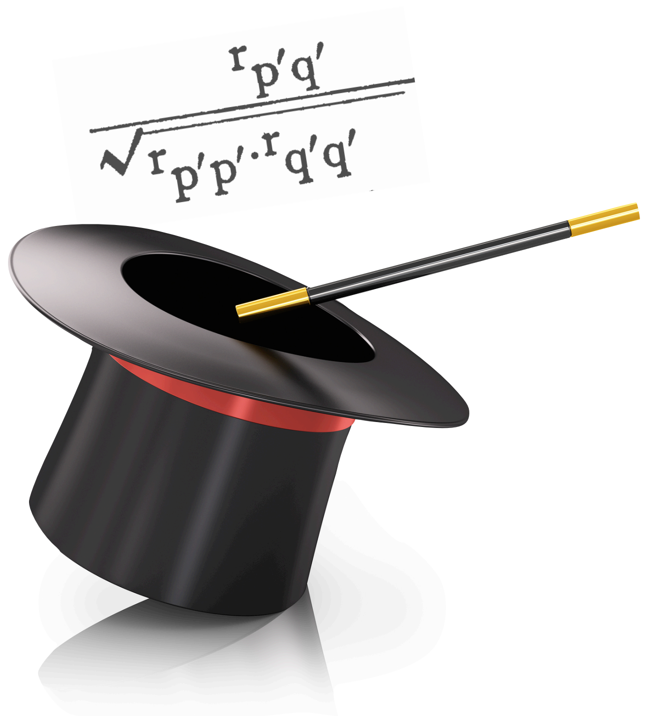
Psicostat

December 20th, 2024

Disattenuation magic:
Learning new tricks from an old method



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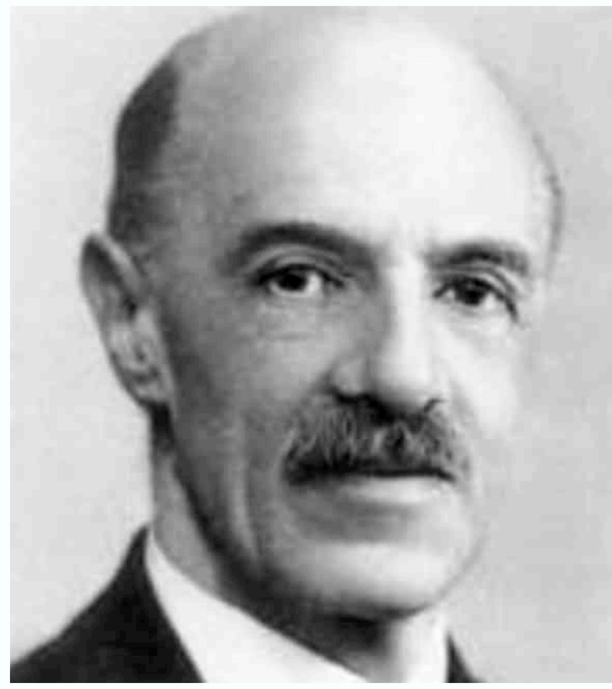
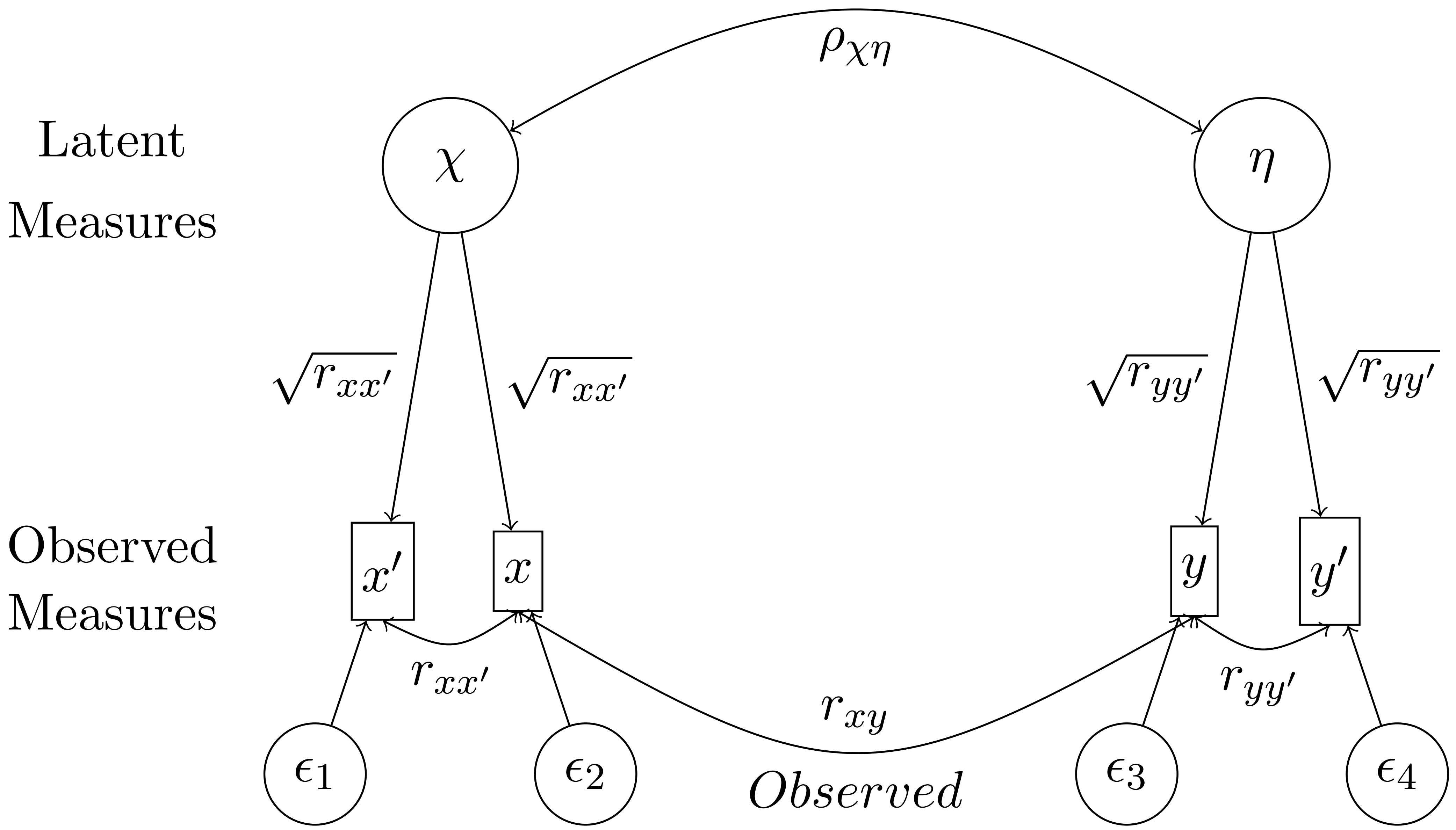
Disattenuation: correcting effect sizes for measurement error



THE PROOF AND MEASUREMENT OF ASSOCIATION BETWEEN TWO THINGS.

By C. Spearman.

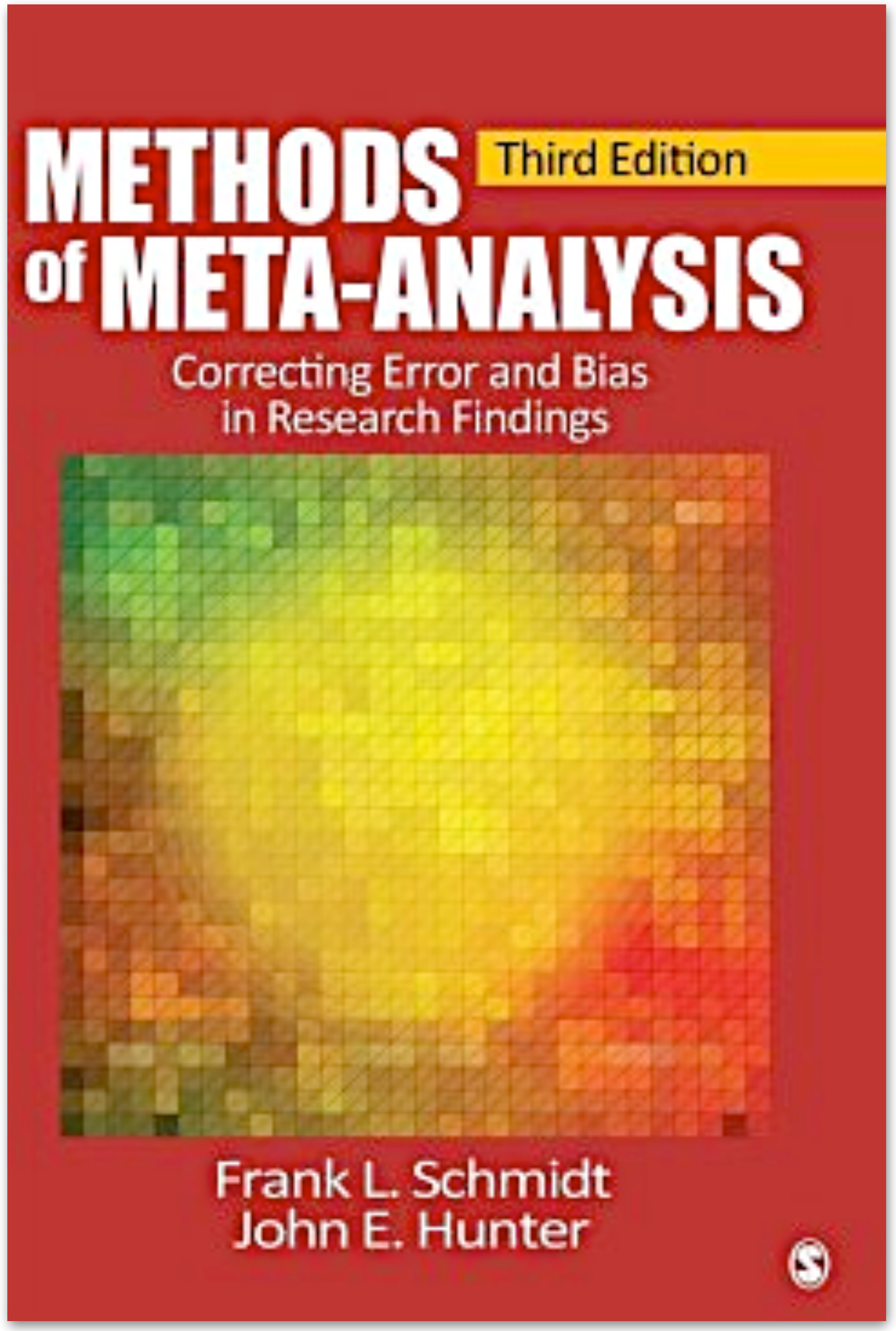
$$r_{pq} = \frac{r_{p'q'}}{\sqrt{r_{p'p'} \cdot r_{q'q'}}}$$



$$r_c = \frac{r_{obs}}{\sqrt{r_{XX}}\sqrt{r_{YY}}}$$

$$d_c = \frac{d_{obs}}{\sqrt{r_{XX}}}$$

$$D_c = \sqrt{\mathbf{d}_c^T \mathbf{R}_c^{-1} \mathbf{d}_c}$$



MULTIVARIATE BEHAVIORAL RESEARCH, 2018
VOL. 53, NO. 4, 481–501
https://doi.org/10.1080/00273171.2018.1457938

Routledge
Taylor & Francis Group

Using Generalizability Theory to Disattenuate Correlation Coefficients for Multiple Sources of Measurement Error

Walter P. Vispoel[a], Carrie A. Morris, and Murat Kilinc

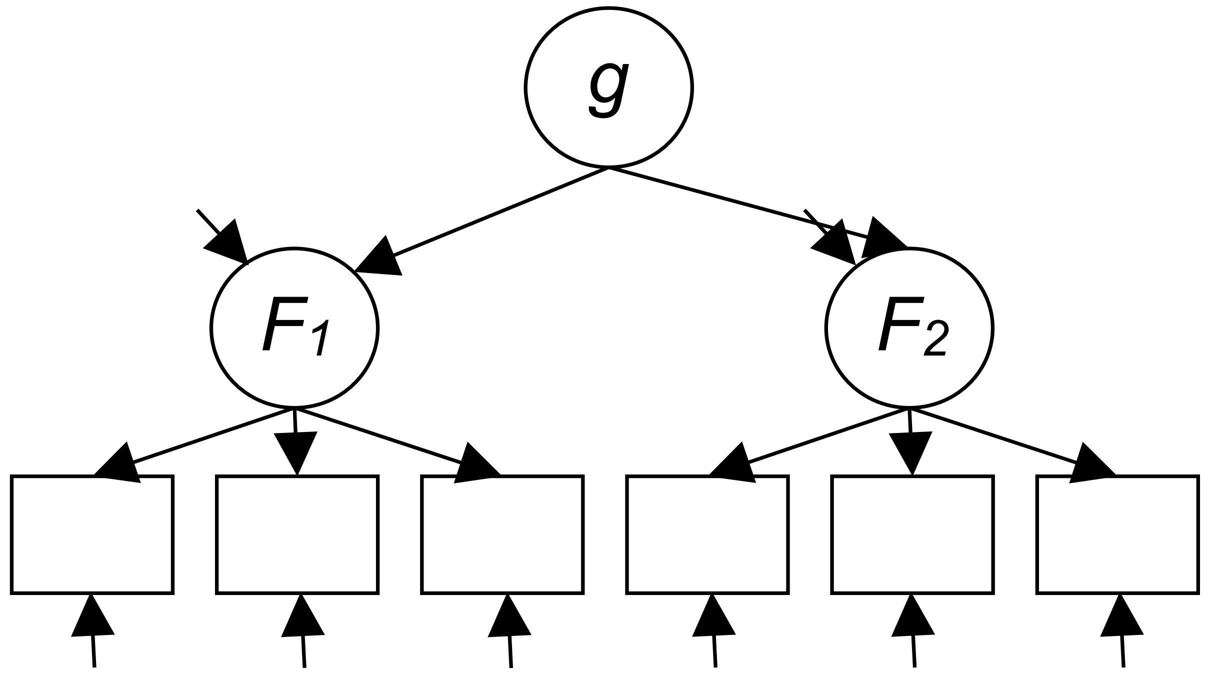
$$\text{Estimated G-coefficient (CES)} = \frac{\hat{\sigma}_p^2}{\hat{\sigma}_p^2 + \left[\frac{\hat{\sigma}_{pt}^2}{n_t} + \frac{\hat{\sigma}_{po}^2}{n_o} + \frac{\hat{\sigma}_{pto,e}^2}{n_t n_o}\right]},$$

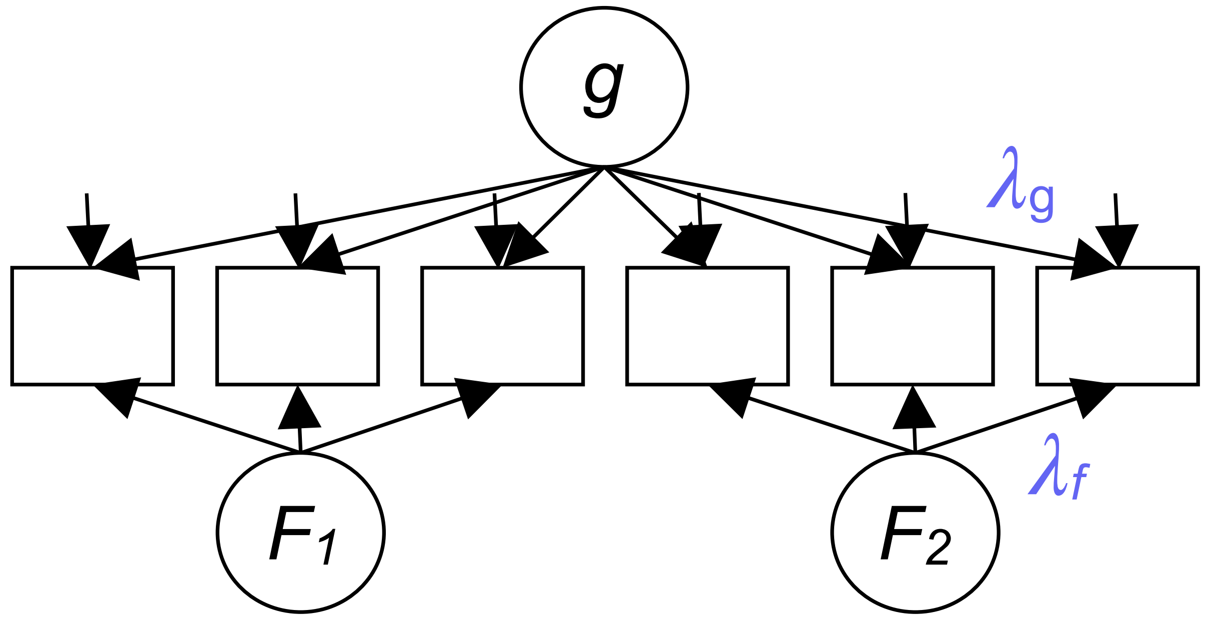
where $\hat{\sigma}^2$ = estimated variance, p = person, t = task (item or split), o = occasion, to = task × occasion interaction, pt = person × task interaction, po = person × occasion interaction, pto,e = person × task × occasion interaction and other error, n_t = number of tasks, and n_o = number of occasions.

Trick #1: g-disattenuation

Omega coefficients: appropriate when a test taps into general + specific factors (McDonald 1999; Zinbarg et al. 2005)

- **hierarchical** or **bifactor** latent structure





Omega-total
(analogous to alpha)

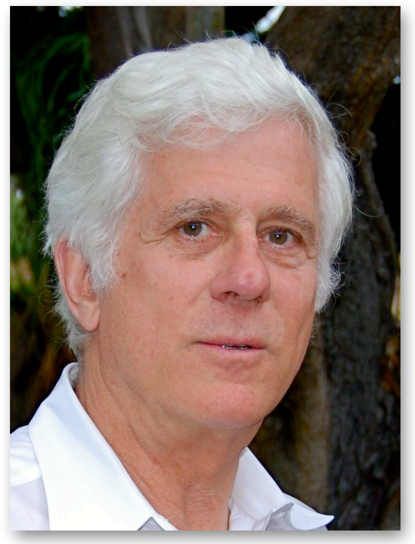
$$\omega_{\mathrm{t}} = \frac{\left(\sum_{i=1}^{k} \lambda_{\mathrm{g}i}\right)^2 + \sum_{f=1}^{n}\left(\sum_{i=1}^{k} \lambda_{fi}\right)^2}{\mathrm{Var}(X)}$$

Omega-general
(omega-hierarchical
ω_{h} as special case)

$$\omega_{\mathrm{g}} = \frac{\left(\sum_{i=1}^{k} \lambda_{\mathrm{g}i}\right)^2}{\mathrm{Var}(X)}$$

Omega-infinity
(AKA limit, asymptotic…)

$$\omega_{\mathrm{inf}} = \frac{\omega_{\mathrm{g}}}{\omega_{\mathrm{t}}} = \frac{\left(\sum_{i=1}^{k} \lambda_{\mathrm{g}i}\right)^2}{\left(\sum_{i=1}^{k} \lambda_{\mathrm{g}i}\right)^2 + \sum_{f=1}^{n}\left(\sum_{i=1}^{k} \lambda_{fi}\right)^2}$$



Reviewer #2 (Bill Revelle): "I would not recommend using ω_h to correct for attenuation. Rather, ω_t or total. While ω_h represents the amount variance associated with general factor of a test, it is not an estimate of how much correlations with other measures are attenuated."

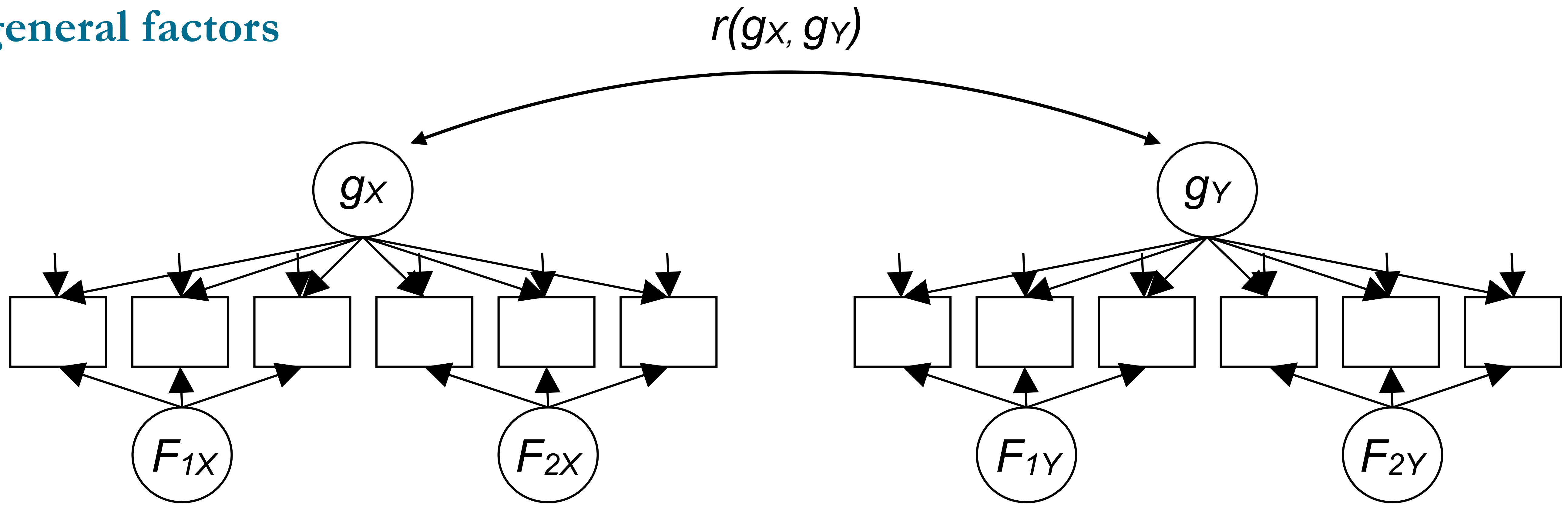
Me: damn, you're right

Me, years later: …wait a minute

→ ω_g (or ω_h) can be used to estimate correlations **between general factors**

$$r_g = \frac{r_{obs}}{\sqrt{\omega_{gX}}\sqrt{\omega_{gY}}}$$

$$SE_g = SE_{obs}\frac{1}{\sqrt{\omega_{gX}}\sqrt{\omega_{gY}}} = SE_{obs}\frac{r_g}{r_{obs}}$$

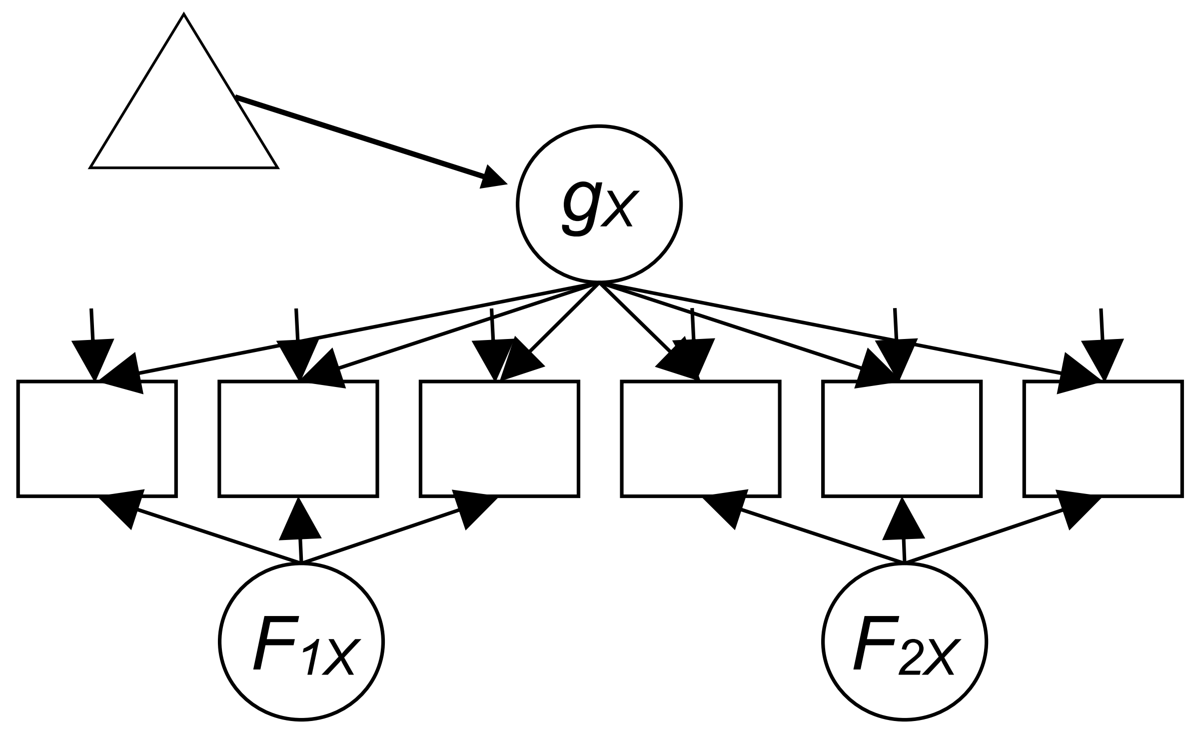


- **Quick, convenient shortcut** that does not require fitting a full SEM; can be calculated from published summary stats

- Key assumption: **uncorrelated secondary factor components** $r(F_X, F_Y) = 0$
  (individual factors may correlate as long as the effects cancel out)

- Easy to calculate **bounds** on the true correlation if violated: $\frac{r_c - \sqrt{1-\omega_{infX}}\sqrt{1-\omega_{infY}}}{\sqrt{\omega_{infX}}\sqrt{\omega_{infY}}} \leq r(g_X, g_Y) \leq \frac{r_c + \sqrt{1-\omega_{infX}}\sqrt{1-\omega_{infY}}}{\sqrt{\omega_{infX}}\sqrt{\omega_{infY}}}$, $\quad r_c = \frac{r_{obs}}{\sqrt{\omega_{tX}}\sqrt{\omega_{tY}}}$

g-disattenuation for standardized mean differences between two groups:



$$d_{\mathrm{g}} = \frac{d_{\mathrm{obs}}}{\sqrt{\omega_{\mathrm{g}X}}}$$

$$SE_{\mathrm{g}} = SE_{\mathrm{obs}} \frac{1}{\sqrt{\omega_{\mathrm{g}X}}} = SE_{\mathrm{obs}} \frac{d_{\mathrm{g}}}{d_{\mathrm{obs}}}$$

- Key assumption: measurement invariance wrt the general factor

Multivariate differences (Mahalanobis' *D*): possible but tricky

- must satisfy assumptions for all r_g's and d_g's
- violations cumulate across variables
- disattenuated correlation matrix more likely to be NPD

Empirical example: sex differences on 16PF personality traits (Kaiser et al. 2020)

	ω_{t}	ω_{h}	ω_{inf}	d_{obs}	d_{c}	d_{g}
A. Warmth	.87	.70	.81	– 0.35	– 0.38	– 0.42
C. Emot. Stability	.89	.74	.83	0.31	0.32	0.36
E. Assertiveness	.87	.72	.83	0.19	0.21	0.23
F. Gregariousness	.85	.47	.55	0.01	0.01	0.01
G. Dutifulness	.90	.73	.82	0.32	0.34	0.38
H. Friendliness	.92	.82	.89	0.00	0.00	0.00
I. Sensitivity	.75	.44	.59	– 0.80	– 0.93	– 1.22
L. Distrust	.89	.74	.83	0.05	0.05	0.06
M. Imagination	.86	.61	.70	0.11	0.12	0.14
N. Reserve	.90	.81	.90	0.15	0.16	0.17
O. Anxiety	.87	.75	.86	– 0.51	– 0.55	– 0.59
Q1. Complexity	.82	.75	.75	– 0.20	– 0.22	– 0.26
Q2. Introversion	.87	.59	.68	– 0.01	– 0.01	– 0.01
Q3. Orderliness	.86	.53	.62	0.09	0.09	0.12
Q4. Emotionality	.85	.60	.70	– 0.09	– 0.10	– 0.11
				D_{obs}	D_{c}	D_{g}
Multivariate				1.18	1.49	2.80

Trick #2: Data matrix disattenuation (DMD)

An error correction method that **directly adjusts the observed variables** to increase their reliability (and yield disattenuated ESs)

- based on the concepts of **matrix whitening** and **coloring**
- exploits the correlational structure of the data + knowledge about the error-free (disattenuated) correlations

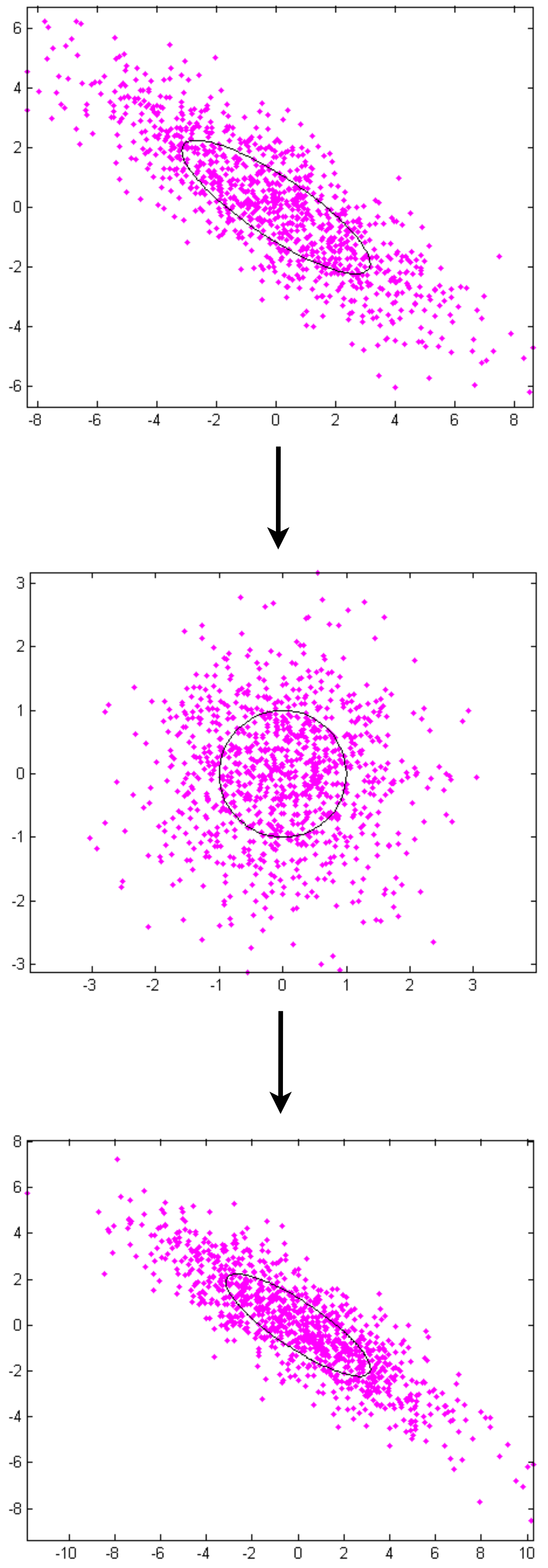
1. Store the means and variances of the observed variables, then standardize $\mathbf{X}_{\text{obs}}$ to yield $\mathbf{Z}_{\text{obs}}$. Calculate the observed correlation matrix $\mathbf{R}_{\text{obs}}$.

2. Use known or estimated reliabilities to obtain the disattenuated correlation matrix $\mathbf{R}_{\text{c}}$ (see Eq. 2). If $\mathbf{R}_{\text{c}}$ is non-positive definite, smooth it to a positive definite matrix (which can be done with numerical methods, e.g., Higham, 2002).

3. Obtain a *whitening matrix* $\mathbf{W}_{\text{obs}}$ from the observed correlation matrix $\mathbf{R}_{\text{obs}}$, and use it to decorrelate the variables in the standardized observed data matrix ($\mathbf{Z}_{\text{obs}}$). The ZCA method is preferred because it maximizes the correlations between the original and whitened variables (Kessy et al., 2018).

4. Obtain a *coloring matrix* $\mathbf{W}_{\text{c}}^{-1}$ from the disattenauted correlation matrix $\mathbf{R}_{\text{c}}$ (a coloring matrix is the inverse of a whitening matrix), and use it to "transfer" the disattenuated correlations onto the whitened data matrix. This yields the standardized corrected data matrix $\mathbf{Z}_{\text{c}}$.

5. If desired, rescale the variables in $\mathbf{Z}_{\text{c}}$ to the original means, with variances adjusted to reflect the correction (more on this below), yielding the unstandardized corrected data matrix $\mathbf{X}_{\text{c}}$.



The main procedure reduces to: $\mathbf{Z}_c = \left(\mathbf{W}_c^{-1}\mathbf{W}_{obs}\mathbf{Z}_{obs}^T\right)^T = \left(\mathbf{R}_c^{1/2}\mathbf{R}_{obs}^{-1/2}\mathbf{Z}_{obs}^T\right)^T$

- "rediscovery" of the **moment reconstruction** technique (Freedman et al. 2004)
- correct **standard errors** can be recovered via bootstrapping (or with simple formulas for correlations, mean differences)

Some simulations results (multivariate normal variables, independent errors; mean true correlation = .20-.28)

1. Disattenuation with known reliabilities

Number of variables

Reliability: observed variables

Reliability: corrected variables

Pop. reliability	N	k = 5	10	20	50	100
ρ_{XX} = .50	100	.53	.55	.58	.62	—
	500	.54	.58	.62	.69	.74
	1,000	.55	.58	.63	.70	.76
	5,000	.55	.58	.63	.72	.78
ρ_{XX} = .70	100	.72	.74	.76	.79	—
	500	.72	.75	.78	.83	.86
	1,000	.72	.75	.78	.83	.87
	5,000	.72	.75	.79	.84	.88
ρ_{XX} = .90	100	.90	.91	.92	.93	—
	500	.90	.91	.92	.94	.95
	1,000	.90	.91	.92	.94	.96
	5,000	.90	.91	.92	.94	.96

2. Disattenuation with approximate reliabilities (+/- .10)

Pop. reliability	N	k = 5	10	20	50	100
ρ_{XX} = .50	100	.53	.55	.58	.62	—
	500	.53	.58	.61	.68	.73
	1,000	.54	.57	.62	.69	.75
	5,000	.54	.58	.63	.71	.77
ρ_{XX} = .70	100	.72	.73	.75	.78	—
	500	.72	.74	.77	.82	.85
	1,000	.72	.75	.78	.82	.86
	5,000	.72	.75	.78	.83	.87
ρ_{XX} = .90	100	.90	.91	.91	.92	—
	500	.90	.91	.92	.93	.94
	1,000	.90	.91	.92	.93	.94
	5,000	.90	.91	.92	.93	.94

→ Knowing the **exact reliabilities** is not critical; rough estimates will do just fine

Comparison with an alternative error correction method: true score imputation (TSI; Mansolf 2023)

DMD

Pop. reliability	*N*	*k* = 5	10	20	50	100
$\rho_{XX} = .50$	100	.53	.55	.58	.62	—
	500	.54	.58	.62	.69	.74
	1,000	.55	.58	.63	.70	.76
	5,000	.55	.58	.63	.72	.78
$\rho_{XX} = .70$	100	.72	.74	.76	.79	—
	500	.72	.75	.78	.83	.86
	1,000	.72	.75	.78	.83	.87
	5,000	.72	.75	.79	.84	.88
$\rho_{XX} = .90$	100	.90	.91	.92	.93	—
	500	.90	.91	.92	.94	.95
	1,000	.90	.91	.92	.94	.96
	5,000	.90	.91	.92	.94	.96

TSI (avg. of 10 imputations)

Pop. reliability	*N*	*k* = 5	10	20
$\rho_{XX} = .50$	100	.48	.51	.47
	1,000	.50	.55	.61
	5,000	.51	.56	.62
$\rho_{XX} = .70$	100	.69	.71	.73
	1,000	.71	.73	.77
	5,000	.70	.74	.78
$\rho_{XX} = .90$	100	.89	.91	.91
	1,000	.90	.90	.92
	5,000	.90	.91	.92

- DMD is somewhat more accurate (esp. in small samples)
- DMD is computationally much, much faster

Behavior Research Methods
https://doi.org/10.3758/s13428-024-02369-5

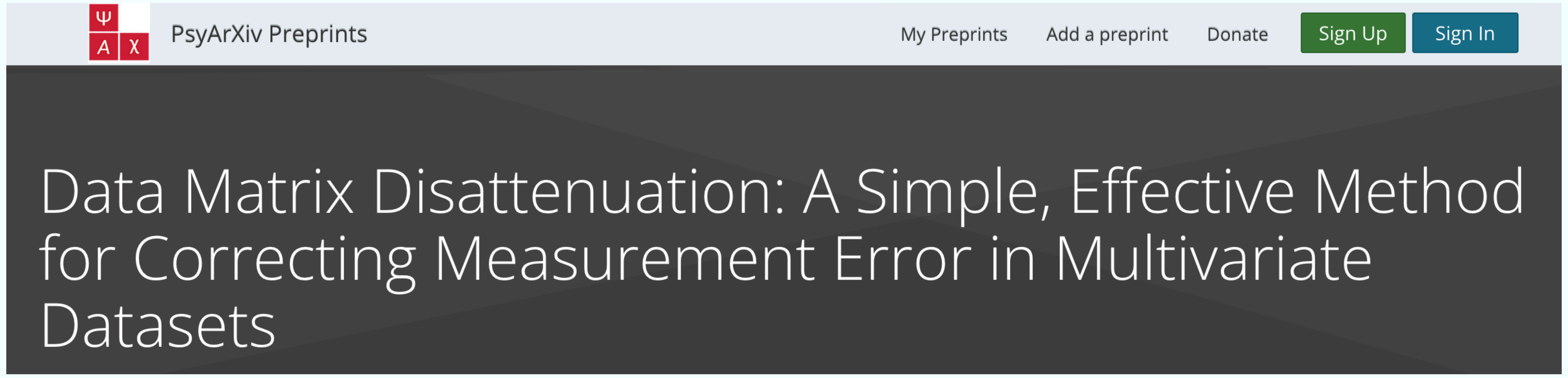
ORIGINAL MANUSCRIPT

Check for updates

Statistical indices of masculinity-femininity: A theoretical and practical framework

Marco Del Giudice[1]

https://osf.io/preprints/psyarxiv/zutrx



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